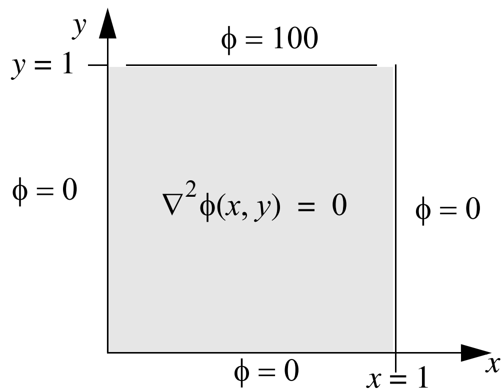
UNIVERSITY OF MANITOBA | **Department of Electrical and Computer Engineering**

24.781 Computational Electromagnetics

ASSIGNMENT 1
Solution of Electrostatic Problems by Finite Difference Methods

September 28, 2005 **Due Date: Monday, October 10, 2005**

A) For the two dimensional rectangular region shown in the figure write a program (any language) to solve for the scalar potential $\phi(x, y)$ on a rectangular grid using Successive Over-Relaxation (SOR).



Use the relative displacement norm, ε, to stop the iterative loop. The relative displacement norm, ε, at iteration (m+1) is derived from the displacement norm δ as:

$$\delta = \sum_{i=1}^{N} \left|\phi_i^{(m+1)} - \phi_i^{(m)}\right| = \|\Delta\phi\|, \; \varepsilon = \frac{\delta}{\|\phi\|} = \delta \Big/ \sum_{i=1}^{N} \left|\phi_i^{(m+1)}\right|$$

where N is the total number of grid points in the two dimensional grid.

B) Using $h = \Delta x = \Delta y = 0.1$ determine the optimum over-relaxation constant ω_{opt}. Choose $\varepsilon = 10^{-6}$ as the stopping condition. Plot number of iterations vs. ω. Plot the potential using a contour plot with 10 level lines between 0 and 100 V. Have your program calculate the total electric flux emanating from the enclosed region by numerically integrating the normal component of the electric field around the boundary. Discuss why this should be so.

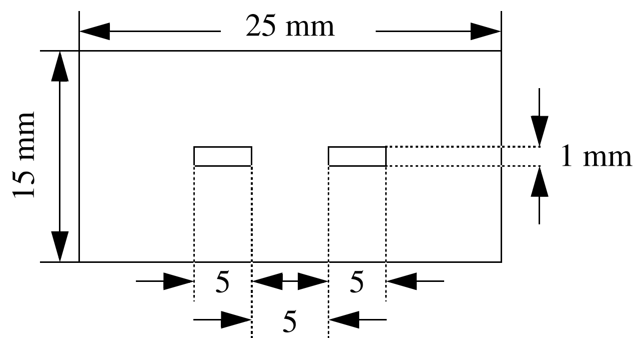
C) Using $h = \Delta x = \Delta y = 0.1$ and the optimum over-relaxation constant ω_{opt} found in part B, plot the required number of iterations for the following relative displacement norms: $\varepsilon = \{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}\}$. What happens when ε is set too low, say 10^{-16}?

D) Decrease h by a factor of 2 three times to $h = \Delta x = \Delta y = \{0.05, 0.025, 0.0125\}$ and set $\varepsilon = 10^{-6}$. Compare the required number of iterations for each h using the optimum over-relaxation constant ω_{opt} found in part B.

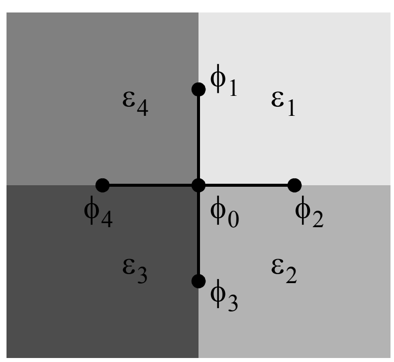
E) Reformulate the problem taking advantage of symmetry across the line defined by $x = 0.5$. What boundary conditions are required for the new problem and what is the finite difference boundary condition you will use? Modify your computer program for this new problem and check that the results are the same.

F) Give a complete derivation for an analytic solution to this problem using eigenfunction expansion and produce contour plot of the difference with the finite difference solution using as many terms in the eigenfunction expansion as required to justify a valid comparison.

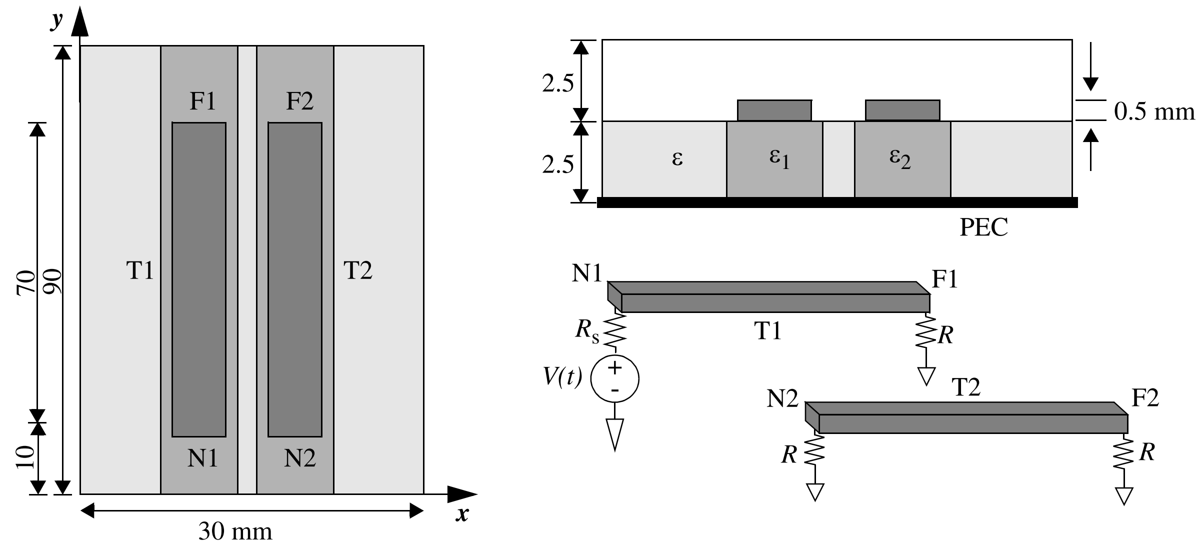
G) Modify your program to solve for the Capacitance Matrix of the transmission line shown in the figure.



H) Derive a finite difference approximation which can be used for the 2-D problem having dielectric discontinuities as shown in the figure. Give the finite difference formula for the computational molecule shown.



I) Modify one of your programs to solve for the capacitance matrix of the transmission line shown in the figure. The width of the striplines is 3 mm and they are spaced 4 mm apart. The dielectric strips are centered under the stripline conductors and are 4 mm wide. Assume that a perfect electric conductor surrounds the striplines completely.



The substrate is made of alumina (*i.e.* a ceramic ε = 10) and the dielectric strips have the following dielectric constant values:

Table 1: Nine Cases of Ceramic Substrate Compensation (ε = 10.0)

Case	1	2	3	4	5	6	7	8	9
ε_1	10.0	10.0	4.7	2.2	2.2	2.2	4.7	4.7	10.0
ε_2	4.7	2.2	10.0	10.0	2.2	4.7	4.7	2.2	10.0